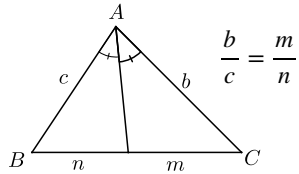
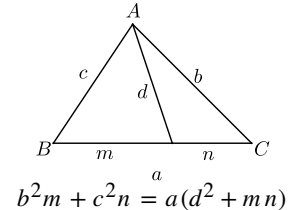


ANGLE BISECTOR THEOREM:



STEWART'S THEOREM:



LOGARITHMS:

$$y = \log_b x \text{ means } b^y = x$$

$$\log_n(ab) = \log_n a + \log_n b$$

$$\log_n\left(\frac{a}{b}\right) = \log_n a - \log_n b$$

$$\log_n(a^k) = k \log_n a$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

QUADRATIC FORMULA:

$$\text{Roots of } ax^2 + bx + c = 0$$

$$\text{are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

AM-GM INEQUALITY:

$$\text{If } a_i \geq 0, \text{ then}$$

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n}$$

SUM AND DIFFERENCE OF POWERS:

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

SUMS OF POWERS OF INTEGERS:

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

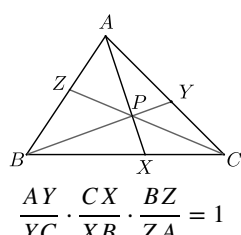
$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

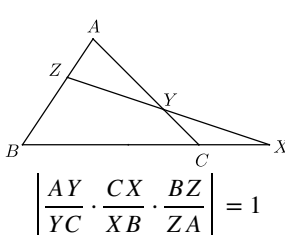
ANALYTIC GEOMETRY OF LINES:

- The line joining (x_1, y_1) to (x_2, y_2) has slope $\frac{y_2 - y_1}{x_2 - x_1}$
- The equation of line through (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$
- The distance from (x_0, y_0) to the line $ax + by + c = 0$ is $\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

CEVA'S THEOREM:



MENELAUS' THEOREM:



MORE TRIANGLE RESULTS:

Law of Sines:

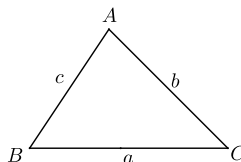
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Heron's Formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \quad \left(s = \frac{a+b+c}{2}\right)$$



SOME 3D GEOMETRY:

- Pyramids and cones with base area B and height h have volume $V = \frac{1}{3}Bh$.
- Volume of a sphere is $V = \frac{4}{3}\pi r^3$
- Surface area of a sphere is $SA = 4\pi r^2$

COUNTING OBJECTS:

Given n distinguishable objects:

- The # of ways to line them up in a row is $n! = n \cdot (n-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1$
- The # of ways to pick a set of r where order matters is ${}_nP_r = \frac{n!}{(n-r)!}$
- The # of ways to pick a set of r where order doesn't matter is $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

BINOMIAL THEOREM:

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \dots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$$

COMPLEX NUMBERS:

Define $i = \sqrt{-1}$, so $i^2 = -1$. Let $z = a + bi$.

Then $\bar{z} = a - bi$ is the complex conjugate of z ,

and $|z| = \sqrt{a^2 + b^2}$ is the modulus of z .

De Moivre's Theorem:

If $z = r(\cos \theta + i \sin \theta)$, then $z^n = r^n(\cos(n\theta) + i \sin(n\theta))$

CYCLIC QUADRILATERALS:

ABCD is Cyclic

$$\iff \angle ADB = \angle ACB$$

$$\iff \angle ABC + \angle ADC = 180^\circ$$

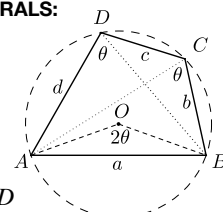
Inscribed Angle Theorem:

$$\angle AOB = 2\angle ADB$$

Ptolemy's Theorem: $ac + bd = AC \cdot BD$

Brahmagupta's Formula:

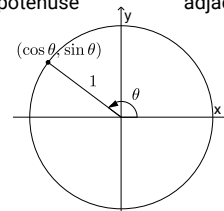
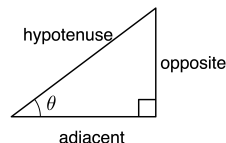
$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}, \quad \left(s = \frac{a+b+c+d}{2}\right)$$



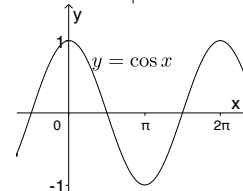
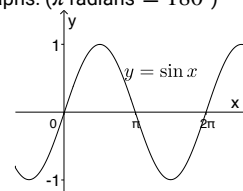
TRIGONOMETRY DEFINITIONS:

Right triangle (SOH-CAH-TOA) & Unit Circle definitions:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



Graphs: (π radians = 180°)



SOME TRIGONOMETRIC IDENTITIES:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \sin^2 \theta + \cos^2 \theta = 1$$

$$\cot \theta = \frac{1}{\tan \theta}, \sec \theta = \frac{1}{\cos \theta}, \csc \theta = \frac{1}{\sin \theta}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A} = \frac{1 - \cos A}{\sin A}$$

MSHSML Formula Sheet

Revised 1/29/2025



ANALYTIC GEOMETRY OF CONIC SECTIONS:

- A **parabola** is the set of points equidistant from a point (the focus) and a line (the directrix)
- A parabola with vertex $(0, 0)$, focus $(0, p)$ and directrix $y = -p$ has equation $x^2 = 4py$.
- An **ellipse** is the set of points whose distances to two fixed points (its foci) add to a constant.
- An ellipse centered at $(0, 0)$ with vertices $(\pm a, 0)$ and covertices $(0, \pm b)$ has area πab and equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Its foci are at $(\pm c, 0)$, where $c^2 = a^2 - b^2$.
- A **hyperbola** is the set of points whose distances to two points (its foci) have a constant difference.
- A hyperbola centered at $(0, 0)$ with vertices $(\pm a, 0)$ and asymptotes $y = \pm \frac{b}{a}x$ has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Its foci are at $(\pm c, 0)$, where $c^2 = a^2 + b^2$.