

**MINNESOTA STATE
HIGH SCHOOL
MATHEMATICS
LEAGUE**

LEAGUE MANUAL

(revised 2026-03-01)

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I. Introduction to the Minnesota State High School Math League

The Minnesota State High School Mathematics League (MSHSML) was founded in 1980-81 by Macalester professor Wayne Roberts and was modeled after leagues that had flourished along the U.S. eastern seaboard since the mid-1940s.

The MSHSML exists to identify students with exceptional interest and/or mathematical ability, give them recognition and encouragement, bring them together with similarly motivated students, and introduce them to topics not commonly taught in the high school curriculum.

Although the MSHSML's focus and related activities tend to draw the attention of students with exceptional ability and interest in mathematics, they are inextricably relevant to educational practices for students at all levels of mathematical aptitude. The MSHSML has always believed that a program for gifted students is shortsighted if it is not developed in a way that strengthens mathematics education for all students.

Currently, the MSHSML administers a season-long competition of multiple meets, open to all qualified students attending schools in Minnesota and schools on the border of Minnesota. The MSHSML invites high-performing students to participate in the American Regions Mathematics League (ARML) and other national contests. We also develop and pursue other math enrichment programs and opportunities for Minnesota students.

II. Competition Rules

A. Eligible Individuals and Teams/Registering a team

The League provides competition for both individuals and teams, consisting of a regular season and culminating in a State Tournament. U.S. high schools in or bordering on the State of Minnesota are eligible to participate. Each school's Math Team is placed in a Division and Section based on criteria outlined below.

Each year, schools need to notify the MSHSML of their intent to participate by submitting a Letter of Intent. Each school is required to pay a fee of \$700 to participate. Payment of the registration fee must be received prior to participation. Schools with Math Team rosters of fewer than 5 students may request a reduced registration fee.

Individual participants must be regularly-enrolled students in a participating high school or a junior high school/middle school in the district of the high school. The number of students that can participate from a school is unlimited.

Students with disabilities may fully participate in the activities of the MSHSML. Coaches of these students should contact the MSHSML office to discuss arrangements for accommodations.

Individual Participation for any student without a Math Team

Homeschooled students or students who attend schools having no official Math Team may fully participate with another school's Math Team unless school policy prohibits such participation. Students may register directly with the MSHSML as individuals and will be assigned to a Division and Section. Students are eligible for all individual awards and honors within that Division/Section and at the state level. Individual students must be accompanied by an adult chaperone at all meets. The cost for an individual student is \$100.

B. Division, Class, and Section Structure

See the Math League website, www.mnmathleague.org, for Division, Class, and Section assignments.

Division Alignment Rules

It is the intention of the board that each Division be composed of at least 5 teams, related either geographically or by connection to an existing athletic conference. If a Division drops below 5 teams, it may operate with fewer teams for 1 year. If the Division does not have at least 5 teams the following year, the teams in the Division may be redistributed to existing Division(s).

Recognizing the difficulty of always finding 5 schools in a geographic area that want to participate, the MSHSML Board passed at its September 2001 meeting the following motion:

“In cases where distance makes it impractical for teams in a Division to come together at one site for each of the five meets of our season, the Division may designate two sites, so long as at least three teams gather at each site. Provisions should be made to have the sites in electronic communication so that teams at each site can see their standing in the meet as each event is graded, and the results should come to the MSHSML office as the report of a single Division.”

Division Alignment Policy

The League has followed these guidelines when forming Divisions:

- Member schools should be in a Division with schools where they are most comfortable.
- Unless prompted by external requests, we attempt to keep intact the same Divisions from year to year.
- When an existing Division is to be changed slightly by addition or deletion of a team or two, this should be done with the cooperation of the Division Coordinator.
- Great effort should be made to have Divisions consist of at least 5 teams.
- When major realignments are undertaken, Division Coordinators teams affected should be invited to a meeting to discuss implementation issues. In all cases, alignment of Divisions worked out through negotiations between League staff and the affected teams must be approved by the League’s Executive Committee.

Division Coordinator

Each Division should, as its last act of business at the conclusion of a season, appoint a Division Coordinator for the following season. It is permissible, even advisable, to have the same person serve as Division Coordinator for several successive years.

The Division Coordinator is a member of the League's Board of Directors and represents the Division at the annual Fall board meeting and at any special meetings of the Board. A Division Coordinator who cannot attend a meeting of the Board should appoint another coach from the Division who then becomes a voting member for that meeting.

Class and Section Structure for State Tournament Qualification

For the purpose of qualifying for the State Tournament, the schools in the League will be organized into three Classes as outlined below. Each Class is composed of 8 Sections with schools placed in a Section based on similar geographic location.

Class Assignment

Placement of teams into Classes shall be based on enrollment numbers from the Minnesota State High School League for the upcoming two-year cycle. Class assignments will be for two-year cycles and are reassigned at the end of the season of odd-numbered years. After a two-year cycle is completed, adjustments in Classes are made based on the averages of the previous two years of performances. Class AA and A teams whose regular-season final score averaged in the top 15 overall in the previous two years are moved to Class AAA for the following two years. Class A teams whose regular-season final score averaged in the top 50 in the regular season in the previous two years are moved to Class AA for the following two years.

Teams that have been moved to a higher Class may be moved back to a lower class if, after a review of the previous two years, the criteria for placement into a higher Class no longer hold. Teams that have been moved to a higher Class will be returned to the Class dictated by the most recently available enrollment numbers if the two-year review dictates it. The teams will be moved if, during the two years they were moved to a higher Class, they did not maintain the standard by which they were moved up. For instance, if a Class AA or Class A team had been moved to Class AAA due to being in the top 15 based on their two-year average, but did not maintain that top 15 average, the team would be placed in the appropriate Class for their performance. This could be Class AA, if a Class A team met the performance criteria for placement in Class AA, or moved back to Class A. If a Class A team was moved to Class AA due to being in the top fifty, but did not maintain the top fifty status for the two years they were moved up, the team would be moved back to Class A.

Any school will have the option to “opt up” to a higher Class by declaring their intent to do so by the Monday four weeks prior to Meet 1 at the beginning of the two-year period. Teams will be notified of approval of their request to opt up by the Monday three weeks prior to Meet 1. Schools who opt up will not displace schools from the Class to which they opt up.

Class placement for schools added to the League in the middle of a Class assignment cycle shall be at the discretion of the Executive Committee or its designee.

C. Regular Season Procedures

Overview of Season Structure and Competition

The regular season of the League consists of 5 meets, gathering within Divisions. The number of students that can attend a meet is unlimited and each student’s score is recorded for individual honors. However, only 8 students, preselected by the coach for each team, have their scores counted toward the total team score (see further explanation below). Individual and team scores are kept for all meets and cumulative scores for individuals and teams are used to determine invitations to the State Tournament. The League recognizes team accomplishments by awarding trophies in each Division and in each Section.

Each meet is organized into two components, Individual Events and a Team Event.

Allowed Materials

Only writing utensils and erasers, scratch paper, a clean copy of the League-created formula sheet, and a copy of the Event provided by the proctor are allowed during Events. In particular, no calculator may be used on any Individual or Team Event. Any additional accommodations must be requested and approved by the League office.

A League-created formula sheet is to be provided to all students. Students may use this formula sheet for practices, but an unaltered version must be provided for meets. It is suggested that coaches make copies of the

formula sheet on colored paper for students to use during meets. The formula sheet will be provided by the League at the state meet. See Appendix D.

Individual Events

Each student may compete in all three Individual Events during any single meet. The Events are labeled A, B, and C but are essentially Algebra topics (event A), Geometry and Trigonometry topics (event B), and Counting, Probability & Statistics, and Number Theory topics (event C). Each event has five problems. All problems are each worth 1 point for a total of 5 points per event. Problems typically increase in difficulty within each event.

In each event, the students are given 15 minutes to solve as many of the five problems as they can. Students are not allowed to communicate with each other during the Individual Events. Topics for these Events vary by meet and are outlined in Appendix B & C. Participants' scores in Individual Events count towards their cumulative season individual scoring totals, regardless of whether or not they are one of the pre-selected scoring students (see Team Event below).

Individual Scoring

Individuals can earn up to 15 points (called the "raw score") in any one meet for a possible total of 75 points for the five-meet season. Individual scores will be weighted using the following formula, with the result called the "Power Score" for an Event:

$$(\text{Power Score}) = (\text{Base Score}) + \ln \frac{1+(\text{number of students attempting the question})}{1+(\text{number of students correctly answering the question})}$$

The Power Scores will be used to rank individuals within their Divisions, Sections, and the state.

The top 50 individual scores (including all ties) are posted within the League's Online Scoring and Reporting System. Cumulative Power Scores are used to determine Division and Section honors and individual invitations to the State Tournament.

Team Event

After the Individual Events are completed, the 8 scoring students (but none of the others who have participated as individuals) compete in a final Team Event consisting of 6 problems across 30 minutes. The 8 scoring team members cooperate with one another in order to produce and submit a single set of answers. Each problem on the Team Event is worth 5 points, for a total of 30 points. It should be noted that non-scoring-team members may also participate in the Team Event for fun in a separate space from the scoring team.

Team Scoring

Since the number of participants is unlimited, larger schools would have an advantage if all student scores were counted for the total team score. To mitigate this advantage, coaches must identify 8 students prior to each meet to be the official scoring members of the team for that meet.

The 8 scoring students can vary from meet to meet, but for any given meet, those 8 students representing their school shall be clearly indicated in the League's Online Scoring and Reporting System roster before the meet begins. Under no circumstances shall a team be allowed to identify its scoring students for a particular meet after the grading of the first Event at that meet has begun. The 8 scoring students' individual scores not only count towards their cumulative season individual scoring total, but also towards the Division, Section, Class, and Statewide rankings of their team.

The team event score plus the sums of individual event scores of the 8 team members determines the official team score for the meet.

The maximum team score for a meet is 150 points and is based on the raw scores of the scoring team members (15-point perfect score for each of the 8 individual scoring team members and a perfect 30-point team event score) for a possible 750 points for the five-meet season.

Team Composition

No more than 6 of the 8 scoring team members shall be beyond the 10th grade.

If all 8 of the scoring team members are beyond the 10th grade then the two students with the highest scores will have their totals removed from the team score. There will be no penalty for the Team Event.

If 7 of the 8 scoring team members are beyond the 10th grade then the one student in grades 11 or 12 with the highest score will have their totals removed from the team score. The score for the eligible student in grades 9 or 10 still counts, even if it is the top score. There will be no penalty for the Team Event.

Ranking of Teams within Divisions and Sections

For the sake of ranking within Divisions and Sections, teams are awarded ordinal ranking points at each meet as follows. If n is the number of schools in the Division or Section, then:

- The 1st place team earns $n + 1$ ordinal points;
- The 2nd place team earns $n - 1$ ordinal points;
- The 3rd place team earns $n - 2$ ordinal points;
- ...and so on until the last place team at the meet earns a minimum of 1 ordinal point.

A team that does not participate in a meet will not receive any points. In case of a tie at a meet, the ordinal points normally awarded for each place are averaged and awarded to each tied team. For example, if there is a 3-way tie for second place, the second-, third- and fourth-place ordinal points are averaged and awarded to each team.

Teams are ranked in their Division and Section by cumulative ordinal points, not overall total season points. If there is a tie in ordinal points after five meets, the Division or Section winner will be the team with the most season points.

Hosting/Running a Meet

All members of a school's team must be physically gathered in the same location to compete at a meet, regardless of the chosen option.

For the first 4 meets ONLY, each Division will have latitude to decide among the following options:

- Option 1: Teams in the Division meet at a common site.
- Option 2: Teams in the Division meet at individual sites.
- Option 3: Some teams in the Division meet at a common site while other teams meet at individual sites.

For Meet 5, Divisions are required to meet in person and conduct meets using paper exams and pencils. Meeting as a Division for Meet 5 is true to the mission of the League, providing an opportunity for students to meet and enjoy mathematics competition in-person. It also facilitates distribution of end-of-season awards.

Each Division Coordinator will submit to the League's Associate Director verification of the teams that attended Meet 5. **All teams are required to meet in person with their division for Meet 5, unless they have received special permission from the Executive Director of the League, or they will be ineligible to participate in the state tournament (as a team or as an individual).**

At all meets, teams may compete by submitting their answers either using paper Events or via entry in the Online Scoring and Reporting System.

Meet Dates and Times

Each meet will be open for competition on the scheduled Monday or the Tuesday immediately following. All Events must be taken consecutively, in a single session, on a single day. Teams competing by submitting answers via the Online Scoring and Reporting System should sign up for a Monday or Tuesday time slot in the Online Scoring and Reporting System.

Prior to Meet 1, each coach should:

- edit team roster, adding and removing students;
- create/verify Student Accounts:
 - Each student must have an account. Go to Team Admin > Student Accounts to generate and retrieve usernames and passwords.
 - Returning students will use the same account as previous years.
- complete a practice Meet with your students so everyone understands the process.

Structure of a Meet at a Common Site

Divisions run meets somewhat differently but often employ a structure resembling the following:

- Convene all students in a central gathering space
- Welcome students, serve refreshments, and remind students of the rules
- Gather graders in a separate room and review problems and solutions
- Verify that all coaches have predetermined their scoring team students
- Announce Individual Event A and direct Event A participants to designated room(s)
- Administer Event A and set up Event B
- Bring Event A student answers to graders
- Post Event A solutions so students can learn solution methods or challenge scoring
- Repeat previous four steps (iv. – vii.) for Events B and C
- Allow students time to review all solutions to Events A, B, and C or challenge scoring
- Announce Team Event and move teams to separate rooms
- Administer Team Event
- Bring teams' Team Event answers to graders
- Post Team Event solutions and allow challenges
- Allow a final 15-minute challenge period, then finalize and report scores

Proctoring

Individual Events

It is suggested that in Individual Events, tests, scratch paper and the formula sheet be laid on desks, face down, before contestants enter the room. It is also suggested that contestants from the same school should not sit next to each other. In some Divisions, contestants write their names and schools on the backs of the exams

before turning them over on the signal to start. The proctor should give a two-minute warning before the end of the event. Contestants should lay their pencils down and turn their papers over when time is called. The proctor collects papers off the desks after contestants leave.

Team Event:

Each team participating in the team event requires a proctor. An individual proctor may be assigned more than one team to proctor.

Coaches' Duties

Before the day of the meet, the Host Coach shall prepare an assignment sheet to be given to each visiting coach upon arrival. This sheet should assign coaches (and perhaps extra people as needed from the host school) to:

- Serve as proctors of Events A & C
- Serve as proctors of Event B (This allows proctors for Event B to set up the room for Event B while A is underway, etc.)
- Serve as graders (2, preferably 3 or more)
- Serve as proctors of teams during team event.
- Monitor waiting areas, supervise distribution of refreshments (if any)

These assignments should leave the host coach free to respond to unexpected requests and generally oversee the meet. The assignment sheet should also give locations for the various events.

Rooms Needed

At each meet a number of rooms are needed as indicated by the following suggestions.

- Large meeting area for students (initial announcements, announcing of events, holding area, posting of solutions and scores, etc.)
- One room or set of rooms for Events A and C; similarly for Event B.
- A room for each team (and for alternate teams if your Division allows them) for use during the team event.
- A room, preferably isolated from areas of activity, for grading. Keep exam materials here during the meet; proctors pick them up as needed. Materials should be turned over so they cannot be read while the meet is in progress.

Displaying Scores

Before the meet, prepare a scoreboard large enough to be seen at some distance. The scoreboard should list team total points accumulated during the season so far. A large marker board will do, but many Divisions use a poster which can then be awarded to the team winning that day's meet so they can display it the next day at their school. Some Divisions use other technology (overheads, monitors) for displaying results.

Copying Materials

Try to have access during the meet to a duplicating machine just in case you run short of materials.

Grading

Individual and team scores need to be compared across the League's many Divisions with awards, scholarships, and appearances in the State Tournament dependent upon these comparisons. Therefore, it is essential that fundamental grading practices be as uniform as possible.

Graders receive League-provided solutions with which to grade student answers.

All answers will be **integers**.

Once the Events are graded, the scores are then entered into the Math League Online Scoring and Reporting System via the League's website.

Students may challenge an answer graded as incorrect by using the Challenge Procedure described below.

Displaying Solutions

Following each Event, copies of the solutions should be displayed for the students to see.

Steps For Every Meet Competing Online

- Meet Setup
 - Go to Team Admin > Team Meet Setup
 - Select the students on your scoring team
- Control Competition
 - Go to Meet Op > Online Comp Control
 - Select who the recorder will be for the scoring team and optionally an alternate recorder.
 - Activate Individual Events (you will need to activate successive rounds)
 - Following each Individual Event, solutions should be displayed for the students to see
 - Activate Team Event
 - Following the Team Event, solutions should be displayed for the students to see
- Verify Grades
 - Go to Meet Op > Grade Online Taken Events
 - Verification may begin as soon as some students have finished their first event
 - Individual Events
 - Select each student and verify auto-scoring was correct.
 - Mark team done with event
 - Team Event
 - Select scoring team and alternates and verify auto-scoring
 - Mark team done with meet

Challenges

If the official solutions contain an error, and it is discovered prior to the meet, all efforts will be made to alert all coaches as to the correct solution. Coaches would then use the corrected solution to judge a response as acceptable or not.

During the meet, if the official solutions contain an error, the coach of the team that discovers the error should immediately notify the League office (see below). If students believe an answer has been graded incorrectly, or if the student had a unique interpretation of the question which resulted in a different answer, the students must submit a challenge to their coach no later than noon on the Thursday **following** the meet.

Note: Most challenges submitted usually fall in one of these two categories – challenges of a grading mistake by the coach (the student actually got it right but the coach mistakenly marked it wrong) or challenges caused by a unique interpretation of the question resulting in a different answer.

Challenges should be handled in this manner:

- Challenges of mistakes made by a coach or grader should be fixed locally.
- All other challenges should be sent to the League office.

Process for submitting a challenge to the League office:

A short explanation as to the rationale for the challenge should be emailed to mathleague@augsborg.edu and to the Division's Coordinator by noon on the Thursday following the meet. Students may write explanations but may need a coach's assistance. The League will make every effort to rule on the challenge by the end of that week. Scores will be adjusted as necessary.

In general, only the students who challenge a solution can be awarded points for that challenge. If a student does not issue a challenge, yet that student's answer is correct in light of the challenge, that student who did not challenge will NOT be awarded the point(s) for a correct solution. Therefore, it is wiser to challenge than not.

Given this challenge process, and to ensure the most uniformity, coaches should mark answers correct ONLY if the student answer matches the answer provided in the League answer key.

Refreshments

The League does not provide funds for meet refreshments. Host coaches should follow practices established within their Division.

Scheduling and Postponement of Meets

Meet dates are selected by the Board of Directors and schools are encouraged to place meets on their calendars as soon as they are published. Postponement or cancellation of a meet because of weather conditions is a decision made at the Divisional level, typically by the designated Division Coordinator, working within any guidelines the Division has established. The League Office shall be notified as soon as possible of any postponement, and the Executive Committee shall, in exercising its right to extend special State Tournament invitations to top-scoring individuals, weigh any possible effects of postponed meets.

Special Circumstances

The League acknowledges that special circumstances, other than weather conditions, may arise that may affect team or individual participation at a meet. Any variation from the standard meet procedures that have been set forth in this manual must obtain prior approval by the Executive Committee or its designee. These variation requests should be sent to the League's Associate Director.

End of Season Honors and Awards

Division Coordinators, working with guidelines developed by Division Coaches, should plan a suitable awards ceremony at the conclusion of the regular season. This most often takes the form of a dinner (or a pizza party) paid for by an area industry (or by assessing each school in the Division). Division Coordinators may forward receipts for their recognition event to the League office for reimbursement. Reimbursement will be made up to \$70 per team which attended Meet 5 in person. Most award ceremonies have been held in conjunction with Meet 5 and include all students who have participated. This format may be changed by any Division wishing to do so,

but it should be remembered that goals of the League are to recognize effort and achievement in mathematics, to give increased visibility to activities available to those with interest and ability in mathematics, and to encourage students with mathematical talent to pursue further training in the discipline. The recognition event should be consistent with those goals. Media coverage is desirable. To assist schools in recognizing students at the end of the year, the Board of Directors has approved the following program of awards for League participants:

- Plaques will be awarded to the first-place team in each Division based on the cumulative ordinal scoring. In Divisions of 8 or fewer teams, a certificate will be awarded to the second-place team. In Divisions of 9-12 teams, certificates will be awarded to the second and third place teams; in Divisions of over 12 teams, certificates will be awarded to the second, third, and fourth place teams. In addition to an engraved statement of achievement (identifying the League, the Division, and the year), each plaque will bear the names of the coach(es) and each school participant who (1) participated as a team member or as an extra in at least three of the five regular season meets, and (2) was selected at least once during the season as a member of the school's scoring team.
- Plaques will similarly be awarded to the first place team in each Section based on cumulative ordinal scoring. Plaques will be engraved following the same guidelines as the plaques for the first place team in each Division.
- Certificates and pins will be awarded to the individual on each team who has over the season accumulated the most points.
- Desktop awards will be given to students in each Division who accumulate the most points. Award items for the top three students in the Division will be selected by the League and change from year to year. All top Division students according to the schedule below will receive pins and certificates. The desktop awards will change from year to year in anticipation that some students may win in successive years, and the number awarded in a Division will vary with the size of the Division as follows:

Teams in Division	Desktop Award	Division Pins	Team Pin	Team Plaques
1-8	3	10	1 per school	1 + 1 certificate
9-12	4	15	1 per school	1 + 2 certificates
Over 12	5	16	1 per school	1 +3 certificates

(Unless the League Associate Director is otherwise instructed by the Division Coordinator, the awards will be sent to the Division Coordinator who should check them beforehand to see that all is in order.)

- Trophies will be awarded to the individuals that finish first, second, and third in individual Power Scoring (overall across the state) during the League's regular five-meet season. These awards will be presented at the State Tournament.
- Certificates will be awarded to any individual who achieves a perfect score on the regular season.

Division Coordinators are not responsible for distributing awards to teams who did not participate in Meet 5 in person (except for those which received special permission from the Executive Director of the League).

Schools are also encouraged to recognize individuals who participate on their Math Team. The awards (a school letter, a pin of some kind) and the method of representation should give recognition to the student and increase

school awareness of the activity. These awards (cost, decision as to who receives them) are completely the responsibility of the local school.

D. State Tournament Procedures

Overview of State Tournament Structure and Competition

The Math League season culminates with a State Tournament. There are three components to the State Tournament: the Invitational Event, in which top-scoring individuals from the regular season compete directly with each other; the Math Bowl, staged as a quick-response event between top scorers in the Invitational; and the Team Contest, which follows the regular season meet rules.

Tournament Invitational Event and Math Bowl

Invitational Event Participants

Invitations to individuals to participate in the Invitational Event will be extended according to the following procedures.

- The top-scoring individual from each Division and each Section shall be invited.
- From the list of top-scoring individuals in the League, ranked on a statewide basis in order of Power Scores earned during the regular season, the top 50 students shall be invited.
- The Executive Committee or its designee may invite other students who because of individual circumstances may not be selected in steps 1 and 2 but who have compiled outstanding individual records.

The Tournament Invitational Event

The Invitational Event is a 30-minute test with a maximum score of 24 points. The event consists of eight “quickie” questions (one point each), four questions intended to be equivalent in difficulty to the higher-numbered questions that normally appear on regular season Individual Events (two points each), and two multiple-part challenge questions (four points each). This is a no-calculator event and all answers will be integers.

The Math Bowl

The Math Bowl is a quick-response elimination competition that operates under the following rules and procedures:

- The top ten scorers in the Invitational Event based on Power Scores are then invited to compete in the Math Bowl. If a student declines to compete in the Math Bowl, the next-highest-ranked student shall be invited to compete. Seasonal scores will be used to break ties.
- The names of the participants will be announced at the time of the event. The participants will be asked to come to the stage and proceed to an assigned seat at a table on stage. They will be asked to print their name and school at the top of a name placard and will be given a small whiteboard on which to write their answers. A dry-erase marker and eraser will be provided.
- This is a no-calculator event.
- Each student will be given a written problem (one problem at a time) with enough space under the problem to do their work. A time limit is imposed on each problem and when time is called, participants hold up their answers. A point is awarded to each student displaying a correct answer.
- At the end of 8 problems the participant(s) with the top number of correct answers (including ties, if any) will continue and the rest of the competitors will leave the stage.

- After each additional question, those who had incorrect answers will be asked to leave the stage. This will continue for as many as 7 more questions. If at the end there are still ties, the Invitational scores will be used to break the ties.
- One winner is declared (if possible). The winner receives a trophy.

Team Contest

Tournament Team Participants

Invitations to teams will be extended according to the following procedure:

- The team with the best ordinal score from each Section is invited to the State Tournament.
- The four highest-ranked teams in each class that did not place first in their Section are invited.

Number of Students per Team at Tournament Team Contest

Invited teams may bring up to eight scoring team members to participate in the Team Contest. Teams wishing to bring one alternate may do so but will be assessed a fee for the alternate. Alternates will be formed into one or more teams to compete; these teams will not be eligible for awards.

Lodging and Additional Fees

Teams which travel more than 50 miles (one way) to the Tournament site will receive one night of lodging provided by the League.

Additional fees will be assessed to teams that need a second night of lodging; teams 50 miles or less from the Tournament site who wish to stay in League-provided lodging; teams requesting extra facilities for preparation (i.e. practice rooms), and teams requesting awards for Assistant Coaches.

Recognition of Individual Scoring Leaders

Students who reach the Invitational Event at the tournament will receive certificates.

The scores received at the Invitational together with the scores received at the tournament will determine the first, second, and third place for Tournament Scoring Leaders. Trophies will be awarded to these individuals.

When scholarship funds are available, scholarships shall be awarded to individuals ranked highest on the basis of the sum of the season total score and the tournament total score. Such awards will be deferred until after high school graduation and sent directly to the student upon League receipt of a letter indicating how the student plans to use the money to further his/her education. If said letter is not received within 3 years of graduation, the student forfeits the scholarship and the monies are returned to the scholarship fund.

Recognition of Top Scoring Teams at the Tournament

Trophies and medallions will be awarded to the top three scoring teams in each Class.

An award for the team that 1) does not earn an award within any Class and 2) demonstrates the most improvement from their average regular season score compared to their State Tournament score will also be presented at the State Tournament. This award is known as the “Quantum Leap” award and is a distinction of significant team improvement.

Tournament Weather Procedures

Due to scheduling commitments for hotel rooms and for the use of the host school, our policy is to hold our tournament on the scheduled day if at all possible. A canceled tournament would be extremely difficult, if not impossible, to reschedule.

If severe weather conditions threaten our ability to proceed with the tournament, information shall be made available via the following:

- on our website: www.mnmathleague.org
- on Twin Cities metropolitan area radio and TV stations
- via email message to participating coaches

The Executive Committee will try to make any determination on the canceling of the tournament by 7 AM on the day of the tournament. Travel conditions typically vary across the state, as do intended modes of travel (bus, van, private automobile). It is expected that participants in the tournament will in all cases follow the rules and directives of responsible officials of their school in deciding whether to attempt the trip to the tournament.

If participants from a school cannot make the trip to the tournament due to weather conditions but can assemble themselves at the same site locally, they may, with approval prior to the Tournament by the League's Executive Committee, participate electronically. Scores earned via this method shall be included with the scores of teams physically present at the meet and shall qualify for awards as if they were present.

For teams and individuals unable to be present at the tournament to accept awards, the League Director or designee shall make a good-faith effort to personally present such awards in an appropriate venue (school awards night, honors banquet, etc.). In cases where several invited schools from the same area of the state cannot get to the tournament, the League shall cooperate with those schools in setting up a suitable recognition event in their area later in the spring.

III. Other League-Sponsored Activities

Minnesota All-State Math Team/American Regions Mathematics League Competition (ARML)

Each spring approximately 80-100 students are invited to be participants on the Minnesota All-State Math Team, representing Minnesota at the national American Regions Mathematics League (ARML) competition. In addition, approximately 10 to 15 students from grades nine and ten are invited to be ARML "students in training." Selection for the All-State Math Team and students in training is based on a combination of a student's individual performance during the regular League season and performance at evaluations during Spring Practices. The top ten scorers in each of these categories are each guaranteed an All-State Math Team invitation.

A student who accepts an invitation to be a member of the Minnesota All-State Math Team has the following responsibilities:

- Attend 3 of four full team practices on Saturdays following the state tournament, including the New Team Member Orientation for new participants.
- Participate at the ARML Competition held at the University of Iowa on the last weekend in May or the first weekend in June.
- Raise \$450 to cover the cost of travel to ARML

Letters of invitation should be sent to the students via their coaches within the week following the State Tournament. A letter will also be sent to the principal of the school attended by each invitee recognizing the honored student and asking for help in raising the necessary funds.

After one or more of the practice sessions, the coaches may decide to invite one or more of the students in training to be members of the All-State Math Team. At that time, a letter will also be sent to the principal of the school attended by each invitee recognizing the honored student and asking for help in raising the necessary funds.

A total of 64 students, four teams of 15 and 4 alternates, will ultimately be selected for the All-State Math Team and travel to Iowa for the competition.

The head coach of the Minnesota All-State Math Team and three or more additional coaches will be selected by the Executive Committee. A stipend will be given to each coach.

In addition, the All State Team will represent Minnesota when possible at national contests throughout the year, including HMMT, CMIMC, PUMAC, BMT, SMT, or other relevant contests.

IV. Resources for Coaching

Coaches' Conference

Each year the League sponsors a conference for coaches with the following goals:

- Give help to coaches in working with mathematically gifted students.
- Create an *esprit de corps* among coaches by coming together to discuss the season just past, possible changes to strengthen the League, and ways coaches work with teams.

A typical conference has work sessions designed to help coaches with the tasks of coaching students gifted in mathematics and with all aspects of preparing them for competition. A social activity for coaches is also part of the conference. We have brought some of the country's best-known mathematics coaches and problem solvers to the conference to present to our coaches.

Problem Database

Coaches are encouraged to use problems from previous years for student practice. Past exams are available for download in the Math League Online Scoring and Reporting System under the "Archive" tab. The online database allows coaches to search for specific topics aligned to the current topic list. (It should be noted that meet topics changed as of the 2024-25 season and may affect searching for problems in the Archive.)

Other Help Topics

Information for coaches on a variety of topics is available both on the League's website and in the Math League Online Scoring and Reporting System.

Appendix A: Officers, Board of Directors, Executive Committee, and Division Coordinators for the 2025-2026 Season

The complete bylaws are available from the League office and are posted on the League's website.

The Board of Directors consists of all Division Coordinators, the three elected League officers, the Executive Director, the Associate Director, the All-State Math Team Coach, the State Tournament Director, the Head Technologist, and the Head of the Problem-Writing Team, as well as any At-Large members elected by the Board. The Executive Committee consists of the three elected League officers, the Executive Director, the Associate Director, and up to five other members of the Board of Directors.

Officers: (3-year term, September 2025 through August 2028)

President: Stacy Paleen, Park High School, Cottage Grove
Secretary: Reid Froiland, Lincoln High School, Thief River Falls
Treasurer: Jenna Innes, accountant, Smith Schafer

Executive Committee: (1-year term, September 2025 through August 2026)

Daniel Honigs, Columbia Heights High School
Gary Kannel, Holy Family Catholic High School, Victoria, Head Technologist
Luke Olson, South St. Paul Secondary School, State Tournament Host
Zach Sheffert, St. Paul Academy and Summit School, All-State Math Team Coach
Colin Gardner-Springer, Head of the Problem-Writing Team

At-Large Board Members: none at this time

Division Coordinators:

Big 9: Kyal Brandt, Rochester Century HS
Canterbury: Gary Kannel, Holy Family Catholic HS
Central Gopher: Ben Thell, St. Cloud Tech HS
Classic Suburban: (vacant)
Dynamic South: Dave Schmiesing, Montevideo HS
Hiawatha: Mike Lacine, Rochester Lourdes HS
Iron Range: Matt Erickson, Hibbing HS
Metro Alliance: Daniel Honigs, Columbia Heights HS
Minneapolis: Jay Selvaag, Minneapolis Washburn HS
Minnesota Valley: Jennifer Floren, Mankato East HS
Mississippi: Curt Michener, Elk River HS
North Suburban: (vacant)
Northern Lights: Reid Froiland, Thief River Falls HS
Polar: David Duesler, Barnum HS

Prairie: Chad Schmiesing, New London-Spicer HS
Rum River: Stephen Larson, Cambridge-Isanti HS
St. Louis River: Matt Solberg, Proctor HS
St. Paul City: Brian Paulson, St. Paul Highland Park HS
South Suburban: Chuck Croatt, Burnsville HS
Southwest Suburban: Kristin Johnson, St. Louis Park HS
Suburban East: Stacy Paleen, Park High School, Cottage Grove
Three Rivers: Allison Kaatz, La Crescent HS
Tri-Metro: Zach Sheffert, St. Paul Academy and Summit School
Twin City Suburban West: Ernie Johnson, Andover HS
Wasioja: Jodi Flynn, Zumbrota-Mazeppa HS
West Central: Logan Knorr, Pelican Rapids HS

Appendix B: Event Topics

A - Algebra	B - Geometry & Trigonometry	C - Counting, Probability & Statistics, Number Theory
1A 1. Decimals, Fractions, and Percents 2. One Variable Linear Equations & Inequalities 3. Exponent Rules 4. Square Roots and Radicals	1B 1. Angles & Angle Relationships 2. Triangle Similarity and Congruence 3. Analytic Geometry of a Straight Line 4. Trigonometry Basics	1C 1. Basic Counting 2. Statistical Measures 3. Prime Factorization & Divisibility Rules 4. GCD and LCM
2A 1. Systems of Linear Equations 2. Binomials and Quadratics 3. Absolute Value 4. The Logarithm	2B 1. Area, Perimeter, and Lengths in Triangles 2. Right Triangles 3. Analytic Geometry of Points and Lines 4. More Elementary Trigonometry	2C 1. Counting Permutations and Independent Events 2. Analyzing Data 3. Basic Probability 4. Base n Arithmetic
3A 1. The Quadratic Formula 2. Polynomials 3. Arithmetic Sequences & Series 4. Complex Number Arithmetic	3B 1. Area, Perimeter, and Lengths in Quadrilaterals & Polygons 2. Problem Solving involving Triangles, Quadrilaterals and Polygons 3. Transformations in the Plane 4. Trigonometric Identities	3C 1. Counting Combinations 2. More Probability 3. Pascal's Triangle and the Binomial Theorem 4. Calculating Digits, especially the Last Digit
4A 1. Optimization Problems 2. Roots of Polynomial Equations 3. Geometric Sequences & Series 4. More Logarithms & Exponents	4B 1. Area, Perimeter, Angles in Circles 2. Lengths involving Circles 3. Analytic Geometry of Circles 4. Law of Sines & Law of Cosines	4C 1. More Counting & Probability 2. Expected Value 3. Remainders & Modular Arithmetic 4. Finding Integer Solutions
5A 1. Non-Linear Equations and Systems of Equations 2. Functional Equations 3. Sums of Powers of Integers 4. General Sequences & Series	5B 1. 3-Dimensional Geometry 2. Geometry Problem Solving 3. Analytic Geometry of Conic Sections 4. Geometry of Complex Numbers	5C 1. Divisor Arithmetic 2. Venn Diagrams & the Principle of Inclusion-Exclusion 3. Geometric Probability 4. More Integer Solutions

Notes:

- Problems may draw on topics from any previous meet.
- Logic or puzzle type problems may appear in any event throughout the season.

Appendix C: Event Topics per Meet

Meet 1 Topics Breakdown

1A. Algebra

- 1A.1. Decimals, Fractions, and Percents**
- Adding, subtracting, multiplying, or dividing fractions and decimals
 - Reducing fractions to lowest terms
 - Converting fractions to decimals
 - Converting decimals (terminating or repeating) to fractions
 - Percent increase & decrease
 - Ratios & Proportions
 - Interest problems
- 1A.2. One Variable Linear Equations & Inequalities**
- Solving linear equations in one variable
 - One variable linear inequalities
 - Word problems leading to linear equations or inequalities
 - Rate problems (distance = rate \times time)
 - Unit Conversion
- 1A.3. Exponent Rules**
- Positive integer exponents
 - $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$
 - $(a^n)^m = a^{nm}$
 - $a^{-n} = \frac{1}{a^n}$, $a^0 = 1$
- 1A.4. Square Roots & Radicals**
- Square roots
 - Cube roots & higher roots
 - Simplifying square roots & radicals of integers
 - Adding, subtracting, multiplying, and dividing radicals
 - Rationalizing simple radical denominators
 - Radicals as exponents: $\sqrt[n]{a} = a^{\frac{1}{n}}$

1B. Geometry & Trigonometry

- 1B.1. Angles & Angle Relationships**
- Angle sums in triangles and polygons
 - Parallel lines and angle relationships
- 1B.2. Triangle Similarity and Congruence**
- Conditions for congruence: SSS, SAS, ASA, AAS
 - Conditions for similarity: AA, SAS, SSS
 - Problem solving in triangles using Similarity and Congruence
- 1B.3. Analytic Geometry of a Straight Line**
- Slope
 - Slope-intercept form of a straight line
 - Point-slope form of a straight line
 - Midpoint of a segment
- 1B.4. Trigonometry Basics**
- Degrees, radians, and converting between the two
 - Right Triangle trig definitions
 - Unit Circle trig definitions

1C. Counting, Prob. & Stats, Num. Theory

- 1C.1. Basic Counting**
- Counting lists
 - Counting by cases
 - Complementary counting
- 1C.2. Statistical Measures**
- Average/mean
 - Median
 - Mode
- 1C.3. Prime Factorization & Divisibility Rules**
- Testing for divisibility by 2 through 12 (except 7)
 - Prime factorization (using divisibility rules and testing primes up to square root)
- 1C.4. GCD and LCM**
- Calculating the Greatest Common Divisor by factoring
 - Calculating the Least Common Multiple by factoring
 - Calculating Greatest Common Divisor using the Euclidean Algorithm
 - $\gcd(a, b) \times \text{lcm}(a, b) = ab$

Meet 2 Topics Breakdown

2A. Algebra

- 2A.1. Systems of Linear Equations**
- Two (or occasionally more) variable linear systems of equations
 - Word problems leading to systems of equations
- 2A.2. Binomials and Quadratics**
- Multiplying binomials
 - Rationalizing denominators using conjugates
 - Difference of squares factorization
 - Factoring a quadratic as a product of binomials
 - Solving quadratic equations by factoring
- 2A.3. Absolute Value**
- Solving absolute value equations and inequalities in one variable
 - Representation on the Number Line
- 2A.4. The Logarithm**
- Definition of logarithm
 - Relationship to exponents
 - $\log_a(ab) = \log_a a + \log_a b$
 - $\log_a\left(\frac{a}{b}\right) = \log_a a - \log_a b$
 - $\log_a(a^x) = k \log_a a$

2B. Geometry & Trigonometry

- 2B.1. Area, Perimeter, and Lengths in Triangles**
- Base-height area formula
 - Medians, angle bisectors, altitudes
 - Triangle inequality
 - Heron's Formula for triangle area
 - Triangle theorems: Angle Bisector, Stewart, Ceva, Menelaus
- 2B.2. Right Triangles**
- The Pythagorean theorem
 - 30-60-90 and 45-45-90 triangles
 - Common-Pythagorean Triples
- 2B.3. Analytic Geometry of Points and Lines**
- Distance between points
 - Finding intersection points of lines
 - Systems of inequalities used to define a region in the plane
 - Areas of polygons on a grid
 - Distance from a point to a line
- 2B.4. More Elementary Trigonometry**
- Trig functions of common angles
 - Basic Identities: $\sin^2 \theta + \cos^2 \theta = 1$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\csc \theta = \frac{1}{\sin \theta}$
 - $\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$, etc.
 - Graphs of trig functions (sin, cos, tan)
 - Inverse trigonometric functions
 - Triangle area using trigonometry: $A = \frac{1}{2} ab \sin C$
 - Solving trigonometric equations

2C. Counting, Probability & Statistics, Number Theory

- 2C.1. Counting Permutations and Combinations**
- The multiplication principle for independent events
 - Counting Permutations: P_r
 - Factorials
 - Knowing when to add and when to multiply
- 2C.2. Analyzing Data**
- Analyzing data in tables
 - Analyzing data in charts
 - Analyzing data in graphs
- 2C.3. Basic Probability**
- Definition of probability
 - Calculating probabilities using basic counting & permutations
- 2C.4. Base n Arithmetic**
- Base n numbers and base n arithmetic
 - Converting base n numbers to and from base 10
 - Converting base n numbers to and from other bases

Meet 3 Topics Breakdown

3A. Algebra

- 3A.1. The Quadratic Formula**
- Solving quadratics by completing the square
 - Using the Quadratic Formula to solve quadratic equations
 - The discriminant and character of roots
 - Quadratic inequalities
- 3A.2. Polynomials**
- Multiplying polynomials
 - Finding integer or rational roots of polynomials (the Rational Root Theorem)
 - Factoring polynomials based on a known root
 - The Remainder Theorem
 - Sum and difference of cubes factorization
 - Sum and difference of odd powers factorization
 - Solving polynomial equations
 - Simplifying rational expressions (including Polynomial Division)
 - Solving rational equations
- 3A.3. Arithmetic Sequences and Series**
- Arithmetic sequence definition
 - Finding the common difference
 - Finding the n th term
 - Arithmetic series definition
 - Calculating the sum of an arithmetic series
 - Arithmetic sequence & series problem solving
- 3A.4. Complex Number Arithmetic**
- Adding and Subtracting complex numbers
 - Multiplying complex numbers
 - The Complex Conjugate
 - Dividing complex numbers

3B. Geometry & Trigonometry

- 3B.1. Area, Perimeter, and Lengths in Quadrilaterals & Polygons**
- Squares, rectangles, parallelograms, the rhombus, trapezoids, & other quadrilaterals
 - Computing area, perimeter, & lengths
 - Polygons (regular and otherwise)
- 3B.2. Problem Solving Involving Triangles, Quadrilaterals and Polygons**
- Using all geometry topics covered to date
 - Specifically covers anything from 1B.1, 1B.2, 2B.1, 2B.2, and 3B.1
- 3B.3. Transformations in the Plane**
- Scaling
 - Reflections, typically across horizontal or vertical lines
 - Rotations: typically by multiples of 90 degrees
 - Relationship between line slope and the tan function
- 3B.4. Trigonometric Identities**
- Angle sum and difference formulas
 - Double angle identities
 - Half angle identities
 - Solving trigonometric equations

3C. Counting, Probability & Statistics, Number Theory

- 3C.1. Counting Combinations**
- Correcting for overcounting
 - Counting combinations: $\binom{n}{r}$
- 3C.2. More Probability**
- Calculating probabilities using combinations & other counting techniques
 - Conditional probability
- 3C.3. Pascal's Triangle and the Binomial Theorem**
- Pascal's Triangle
 - The Binomial Theorem: coefficients of $(x + y)^n$
- 3C.4. Calculating Digits, especially the Last Digit**
- Finding the last digit of a sum, product, or power of integers
 - Counting trailing zeros of products of integers

Meet 4 Topics Breakdown

4A. Algebra

- 4A.1. Optimization Problems**
- Minimum or maximum value of quadratic expressions (e.g. by completing the square)
 - The Arithmetic-Geometric mean inequality
- 4A.2. Roots of Polynomial Equations**
- Vieta's formula for the sum of roots of a polynomial
 - Vieta's formula for the product of roots of a polynomial
 - Vieta's formula for relationships between roots and other coefficients of a polynomial
- 4A.3. Geometric Sequences and Series**
- Geometric sequence definition
 - Finding the common ratio
 - Finding the n th term
 - Geometric series definition
 - Calculating the sum of a geometric series
 - Geometric sequence & series problem solving
- 4A.4. More Logarithms & Exponents**
- The change of base formula: $\log_a b = \frac{\log_c b}{\log_c a}$
 - Solving exponential equations
 - Solving logarithmic equations

4B. Geometry & Trigonometry

- 4B.1. Area, Perimeter, and Angles in Circles**
- Sectors
 - Circular segments
 - Central & inscribed angles
 - The Inscribed Angle Theorem
- 4B.2. Lengths Involving Circles**
- Internal and External tangent lines
 - Power of a Point
 - Cyclic quadrilaterals
 - Angle relationships in cyclic quadrilaterals (e.g. opposite angles)
- 4B.3. Analytic Geometry of Circles**
- Circle equations
 - Intersections between circles
- 4B.4. Law of Sines & Law of Cosines**
- The Law of Sines
 - The Law of Cosines
 - Finding lengths and angles in triangles and other geometric figures using these laws

4C. Counting, Probability & Statistics, Number Theory

- 4C.1. More Counting & Probability**
- Counting integer solutions to $x_1 + x_2 + \dots + x_k = k$ ("sticks and stones"), including positive and non-negative cases
 - Counting more than two groups (multinomials)
 - Counting paths in a grid
 - Counting & probability problems using all topics to date
- 4C.2. Expected Value**
- Definition based on individual probabilities
 - Expected value problem solving
- 4C.3. Remainders & Modular Arithmetic**
- Calculating remainders
 - Modular arithmetic notation
 - Remainders of sums, products, and powers of integers
- 4C.4. Finding Integer Solutions**
- Finding integer solutions by factoring & casework
 - Finding integer solutions by completing the rectangle ("Simon's Favorite Factoring Trick")

Meet 5 Topics Breakdown

5A. Algebra

- 5A.1. Non-Linear Equations and Systems of Equations**
- Solving radical equations
 - Identifying extraneous roots
 - Solving more complex equations or systems of equations (combining any techniques used to date)
- 5A.2. Functional Equations**
- Solving for a function based on given properties of the function
 - Finding function value for a specific input given properties of the function
 - Domain and Range; Function Composition; Inverse Functions
 - Functional Operations (Invented operators)
- 5A.3. Sums of Powers of Integers**
- Formula for $1 + 2 + \dots + n$
 - Formula for $1^2 + 2^2 + \dots + n^2$
 - Formula for $1^3 + 2^3 + \dots + n^3$
 - Finding related sums using these formulae
- 5A.4. General Sequences & Series**
- The Fibonacci sequence
 - Recursively defined sequences
 - Telescoping sums
 - General sequence & series problem solving

5B. Geometry & Trigonometry

- 5B.1. 3-Dimensional Geometry**
- Triangular and polygonal prisms & pyramids, cylinders, cones, spheres
 - Calculating volume and surface area
- 5B.2. Geometry Problem Solving**
- Problem solving using all Geometry topics
 - Could include any topics from subtopics 1 and 2 of previous 5 events
 - Specifically topics 1B.1, 1B.2, 2B.1, 2B.2, 3B.1, 3B.2, 4B.1, 4B.2
- 5B.3. Analytic Geometry of Conic Sections**
- Axis of symmetry
 - Focus-directrix definition of a parabola
 - Parabola equations
 - Focus-focus definition of an ellipse
 - Ellipse equations
 - Focus-focus definition of a hyperbola
 - Hyperbola equations
- 5B.4. Geometry of Complex Numbers**
- The Complex Plane
 - Modulus of Complex Numbers
 - Distance in the Complex Plane
 - Polar representation of a complex number, relationship to trigonometry
 - De Moivre's Theorem

5C. Counting, Probability & Statistics, Number Theory

- 5C.1. Divisor Arithmetic**
- Counting divisors of a number
 - Sum of divisors of a number
- 5C.2. Venn Diagrams & the Principle of Inclusion-Exclusion**
- Counting with Venn diagrams (2 or 3 sets)
 - Counting using the principle of inclusion-exclusion (any number of sets)
- 5C.3. Geometric Probability**
- Calculating probability as a fractional area
- 5C.4. More Integer Solutions**
- More general diophantine equations
 - Solving modular equations and systems

Appendix D: Formula Sheet

ANGLE BISECTOR THEOREM:

$$\frac{b}{c} = \frac{m}{n}$$

CEVA'S THEOREM:

$$\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} = 1$$

MENELAUS' THEOREM:

$$\left| \frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} \right| = 1$$

CYCLIC QUADRILATERALS:

ABCD is Cyclic

$$\Leftrightarrow \angle ADB = \angle ACB$$

$$\Leftrightarrow \angle ABC + \angle ADC = 180^\circ$$

Inscribed Angle Theorem:

$$\angle AOB = 2\angle ADB$$

Ptolemy's Theorem: $ac + bd = AC \cdot BD$

Brahmagupta's Formula:

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}, \quad (s = \frac{a+b+c+d}{2})$$

STEWART'S THEOREM:

$$b^2m + c^2n = a(d^2 + mn)$$

MORE TRIANGLE RESULTS:

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Heron's Formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \quad (s = \frac{a+b+c}{2})$$

TRIGONOMETRY DEFINITIONS:

Right triangle (SOH-CAH-TOA) & Unit Circle definitions:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Graphs: (π radians = 180°)

LOGARITHMS:

$y = \log_b x$ means $b^y = x$

$$\log_n(ab) = \log_n a + \log_n b$$

$$\log_n\left(\frac{a}{b}\right) = \log_n a - \log_n b$$

$$\log_n(a^k) = k \log_n a$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

SOME 3D GEOMETRY:

- Pyramids and cones with base area B and height h have volume $V = \frac{1}{3}Bh$.
- Volume of a sphere is $V = \frac{4}{3}\pi r^3$
- Surface area of a sphere is $SA = 4\pi r^2$

QUADRATIC FORMULA:

Roots of $ax^2 + bx + c = 0$

$$\text{are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

COUNTING OBJECTS:

Given n distinguishable objects:

- The # of ways to line them up in a row is $n! = n \cdot (n-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1$
- The # of ways to pick a set of r where order matters is $nPr = \frac{n!}{(n-r)!}$
- The # of ways to pick a set of r where order doesn't matter is $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

SOME TRIGONOMETRIC IDENTITIES:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sin^2 \theta + \cos^2 \theta = 1$$

$$\cot \theta = \frac{1}{\tan \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A} = \frac{1 - \cos A}{\sin A}$$

AM-GM INEQUALITY:

If $a_i \geq 0$, then

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n}$$

BINOMIAL THEOREM:

$$(x + y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \dots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$$

SUM AND DIFFERENCE OF POWERS:

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

COMPLEX NUMBERS:

Define $i = \sqrt{-1}$, so $i^2 = -1$. Let $z = a + bi$.

Then $\bar{z} = a - bi$ is the complex conjugate of z ,

and $|z| = \sqrt{a^2 + b^2}$ is the modulus of z .

De Moivre's Theorem:

If $z = r(\cos \theta + i \sin \theta)$, then $z^n = r^n(\cos(n\theta) + i \sin(n\theta))$

SUMS OF POWERS OF INTEGERS:

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

ANALYTIC GEOMETRY OF LINES:

- The line joining (x_1, y_1) to (x_2, y_2) has slope $\frac{y_2 - y_1}{x_2 - x_1}$
- The equation of line through (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$
- The distance from (x_0, y_0) to the line $ax + by + c = 0$ is $\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

ANALYTIC GEOMETRY OF CONIC SECTIONS:

- A **parabola** is the set of points equidistant from a point (the focus) and a line (the directrix)
- A parabola with vertex $(0, 0)$, focus $(0, p)$ and directrix $y = -p$ has equation $y = 4px^2$.
- An **ellipse** is the set of points whose distances to two fixed points (its foci) add to a constant.
- An ellipse centered at $(0, 0)$ with vertices $(\pm a, 0)$ and covertices $(0, \pm b)$ has area πab and equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Its foci are at $(\pm c, 0)$, where $c^2 = a^2 - b^2$.
- A **hyperbola** is the set of points whose distances to two points (its foci) have a constant difference.
- A hyperbola centered at $(0, 0)$ with vertices $(\pm a, 0)$ and asymptotes $y = \pm \frac{b}{a}x$ has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Its foci are at $(\pm c, 0)$, where $c^2 = a^2 + b^2$.

2024-25 Formula Sheet

